PROBLEMS.

19. Proposed by MISS LECTA MILLER, B. L., Professor of Natural Science and Art, Kidder Institute, Kidder, Missouri.

Bought sugar at $6\frac{1}{2}$ cents a pound, waste by transportation and retailing was 5%; interest on first cost to time of sale was 2%. How much must be asked per pound to gain 25%?

20. Proposed by L. B. HAYWARD, Superintendent of Schools, Bingham, Ohio.

I owed a merchant $600. The merchant agreed to take part of the amount and wait a year for the balance, if I would pay interest in advance. I paid $300. How much of this was interest on the unpaid balance, and how much went toward the payment of the debt?

21. Proposed by A. L. FOOTE, No 80, Broad St., New York City.

A merchant bought a certain quantity of corn for which he paid a certain sum of money; but on measuring he found only $\frac{4}{4}\frac{8}{0}$ of the quantity he expected. He sold it gaining $\frac{1}{8}$ of the cost and received $2,160, which was at the rate of $12\frac{4}{13}$ cents per bushel more than he would have paid had he received the quantity expected. How many bushels did he suppose he had bought, and at what price?

[Selected from *Robinson's Arithmetical Problems.*]

[Solutions to the e problems should be received on or before June 1st.]

ALGEBRA.

Conducted by J. M .COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

9. Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

$$\left.\begin{array}{l} x+y^2+z^3=21 \\ x^2+y^3+z=45 \\ x^3+y+z^2=71 \end{array}\right\} . \text{ Find } x, y, \text{ and } z.$$

Solution by the Proposer.

The equations may be written, as follows:

$$x-4+(y-3)(y+3)=(2-z)\ (4+2z+z^2)\ldots\ldots(1)$$
$$(x-4)(x+4)+(y-3)(9+3y+y^2)=2-z\ldots\ldots(2)$$
$$(x-4)(16+4x+x^2)+y-3=(2-z)(2+z)\ldots\ldots(3)$$

Let $16+4x+x^2=a$, $9+3y+y^2=b$, $4+2z+z^2=c$, $x+4=d$, $2+z=e$. Then from (1), (2), and (3), we get $x-4=c(2-z)-(y^2-9)\ldots\ldots(4)$,

$$x-4=\frac{2-z-b(y-3)}{d}\ldots\ldots(5),\quad x-4=\frac{e(2-z)-(y-3)}{a}\ldots\ldots(6).$$

Eliminating $(x-4)$ between (4) and (6), and (5) and (6), we get, $(ac-e)(2-z)=a(y^2-9)-(y-3)\ldots\ldots(7)$, $(a-de)(2-z)=(ab-d)(y-3)\ldots.(8)$. Eliminating $(2-z)$ between (7) and (8), we get, $a(y^2-9)(a-de)=a(y-3)(1+abc-dc-be)$, or $(y^2-9)\ A=(y-3)B$, suppose.

$$\therefore\ y^2-9=y\frac{B}{A}-3\frac{B}{A}\ ,\ \therefore\ y^2-\frac{B}{A}y=9-3\frac{B}{A}.$$

Completing the square root, $y^2-\frac{B}{A}y+\frac{B^2}{4A^2}=9-3\ \frac{B}{A}+\frac{B^2}{4A^2},\ \therefore\ y=3.$

Substituting value of y in (4) and (5),

$$x-4=c(2-z)\ldots.(9),\ d(x-4)=2-z\ldots.(10).$$

Eliminating $(2-z)$ between (9) and (10), $\frac{x-4}{c}=d(x-4)=x^2-16,$

$$\therefore\ x^2-\frac{x}{c}=16-\frac{4}{c}.\quad \therefore\ x^2-\frac{x}{c}+\frac{1}{4c^2}=16-\frac{4}{c}+\frac{1}{4c^2},\ \therefore\ x=4.$$

Values of x and y in $x^2+y^3+z=45$, gives $z=2$.

$\therefore\ x=4,\ y=3,\ z=2.$

Also solved by *Professor W. F. Bradbury.*

10. Proposed by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania.

$$x^2+y^2+w^2+z^2=65\ldots.(1),$$
$$(x+z)^2+(y+w)^2=113\ldots.(2),$$
$$(y+z)^2+(x+w)^2=117\ldots.(3),$$
$$(x+y)^2+(z+w)^2=125\ldots.(4).$$

How many values has each of the four unknown quantities?

Solution by W. F. BRADBURY, A. M., Head-Master Cambridge Latin School, Cambridge, Massachusetts.

From (1) subtract (2), (3), and (4), successively,

$$2xz+2yw=48\ldots.(5),$$
$$2yz+2xw=52\ldots.(6),$$
$$2xy+2zw=60\ldots.(7).$$

Adding (5), (6), (7), and (1), we get,

$$(x+y+z+w)^2=225\ldots.(8),\quad x+y+z+w=\pm15\ldots.(9),$$

Using only + values, $x+z=15-(y+w)\ldots.(10)$.

Substituting in (2), $225-30(y+w)+(y+w)^2+(y+w)^2=113\ldots.(11)$, $(y+w)^2-15(y+w)=-56\ldots.(12)$,

$$y+w=\frac{15}{2}\pm\sqrt{\frac{225}{4}-\frac{224}{4}}=\frac{15}{2}\pm\frac{1}{2}=8,\text{ or }7\ldots.(13).$$

Hence from (10), $x+z=7$, or 8. In like manner, substituting from (9) in (3) and (4), we find $y+z=6$, or 9; $x+w=9$, or 6; $x+y=5$; $z+w=10$.

From these we find, $x=3$, or 2, $y=2$, or 3, $w=6$, or 4, $z=4$, or 6. Using the negative values other answers can be found.

[There are in all 16 values for each of the unknown quantities, arising from the reduced equations $x+y+z+w=\pm15$, $x+y+z+w=\pm5$, $x+y+z-w=\pm1$, $x-y-z+w=\pm3$, as follows:

$x=\pm6,\ \pm4,\ \pm2,\ \pm3,\ \pm4\frac{1}{2},\ \pm5\frac{1}{2},\ \pm3\frac{1}{2},\ \pm1\frac{1}{2}.$
$y=\pm4;\ \pm6,\ \pm3,\ \pm2,\ \pm5\frac{1}{2},\ \pm4\frac{1}{2},\ \pm1\frac{1}{2},\ \pm3\frac{1}{2}.$
$z=\pm2,\ \pm3,\ \pm6,\ \pm4,\ \pm3\frac{1}{2},\ \pm1\frac{1}{2},\ \pm4\frac{1}{2},\ \pm5\frac{1}{2}.$
$w=\pm3,\ \pm2,\ \pm4,\ \pm6,\ \pm1\frac{1}{2},\ \pm3\frac{1}{2},\ \pm5\frac{1}{2},\ \pm4\frac{1}{2}.$—EDITOR.]

Also solved by Professor *G. B. M. ZERR.*